Date 28/07/2020

Dy- Mathematics

Topic - Partial differential Eqⁿ

By Dr. Ram Anek yadav Deptt. of Maths, Marwari College, Darbhanga

(1) Working rule for using charpit's method.

(a) Shift all the terms of given eqⁿ to the left side and denote the left side by $F(x,y,z,p,q)$.

(b) Find $\frac{\partial F}{\partial x}$, $\frac{\partial F}{\partial y}$, $\frac{\partial F}{\partial z}$, $\frac{\partial F}{\partial p}$, $\frac{\partial F}{\partial q}$.

(c) Write the charpits auxiliary eqⁿ and substitute the values of Partial derivatives of f and simplify

(d) Select any two fractions so that the resulting integral is the simplest relation involving at least one of p and q. This relation and given eqⁿ are solved to find p & q.

(d) Put the values of p and q in the eqn $dz = p\,dx + q\,dy$ and integrate. This gives the complete solution of the given equation.

eg. Solve $2zx - px^2 - 2qxy + pq = 0$

Solution: Here $F(x, y, z, p, q) \equiv 2zx - px^2 - 2qxy + pq = 0$

Hence $\frac{\partial F}{\partial p} = -2x + q$, $\frac{\partial F}{\partial q} = -2xy + p$, $\frac{\partial F}{\partial z} = 2x$

$\frac{\partial F}{\partial x} = 2z - 2px - 2qy$ & $\frac{\partial F}{\partial y} = -2qx$

Auxiliary equations are

$$\frac{dp}{\frac{\partial F}{\partial x} + p\frac{\partial F}{\partial z}} = \frac{dq}{\frac{\partial F}{\partial y} + q\frac{\partial F}{\partial z}} = \cdots = \cdots$$

$$\Rightarrow \frac{dp}{(2z - 2px) - 2qy + p(2xy)} = \frac{dq}{-2qx + q(2xy)} = \cdots = \cdots$$

$$\Rightarrow \frac{dp}{2z - 2qy} = \frac{dy}{0} = \cdots$$

Taking the first two ratio, we have

$$\frac{dp}{2z - 2qy} = \frac{dq}{0}$$

Integrating, we get

$dq = 0 \Rightarrow q = a$